Final state interaction in kaons decays and $\pi\pi$ scattering lengths measurements.

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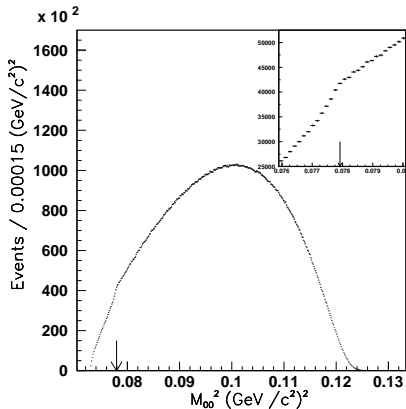
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Outline

1. Cusp in $K^+ \to \pi^0 \pi^0 \pi^+$ decay.
2. Scattering lengths as a test of chiral symmetry breaking.
3. Electromagnetic effects.
4. Scattering lengths extraction from $K^\pm \to \pi^+ \pi^- e^\pm \nu$ (K_{e4}) decay.

Cusp in $K^+ \rightarrow \pi^0\pi^0\pi^+$ decay.

J.Batley et al.,Phys.Lett. ,2006 Experiment NA48/2, SPS,CERN, 2003-2004, 6×10^7; $K^\pm \rightarrow \pi^0\pi^0\pi^\pm$
Anomaly (cusp) in $\pi^0\pi^0$ invariant mass distribution at $M^2 = 4m_c^2$

Two step decay in $K^+ \rightarrow \pi^+ \pi^0 \pi^0$

N.Cabibbo, Phys. Rev. Lett., 2004 Direct $K^+ \rightarrow \pi^+ \pi^0 \pi^0$;
Two step contribution as a result of charge exchange in decay
$K^+ \rightarrow \pi^+ \pi^+ \pi^-; \pi^+ \pi^- \rightarrow \pi^0 \pi^0$

$$T = T_0 + 2ia_x k T_+; \quad a_x(\pi^+ \pi^- \rightarrow \pi^0 \pi^0) = \frac{a_0 - a_2}{3}$$

$T_0(K^+ \rightarrow \pi^+ \pi^0 \pi^0); \quad T_+(K^+ \rightarrow \pi^+ \pi^- \pi^+)$ -unperturbed real
amplitudes (without final state interaction);
a_0, a_2 are S-wave $\pi\pi$ scattering lengths in the isospin I=0 and
I=2 states; $k = \frac{1}{2}\sqrt{M^2 - 4m_c^2}$ - the momenta of charged
pions in charge exchange reaction $\pi^+ + \pi^- \rightarrow \pi^0 + \pi^0$.
Under charged pions threshold it becomes imaginary $k = i\kappa$

$$|T|^2 = T_0^2 + 4\frac{(a_0 - a_2)^2 k^2}{9} T_+^2; M^2 > 4m_c^2$$

$$|T|^2 = T_0^2 + 4\frac{(a_0 - a_2)^2 \kappa^2}{9} T_+^2 - 4\frac{(a_0 - a_2)\kappa}{3} T_0 T_+; M^2 < 4m_c^2$$

Above threshold $M^2 > 4m_c^2$ decay rate $\sim a^2$.

V.N. Gribov Nucl.Phys. (1958) $\tau^+ \to 2\pi^+ + \pi^-$ $\tau' \to 2\pi^0\pi^+$

N.Cabibbo: Under threshold $M^2 < 4m_c^2$ decay rate $\sim a_0 - a_2$

$a_0 = 0.22; a_2 = -0.044$ small quantities.

Scattering lengths as a test of chiral symmetry breaking.

Chiral Perturbation Theory (S.Weinberg,1966) predicts the values of scattering lengths $a_0, a_2 (I = 0, 2)$ with high precision. Weinberg result : $a_0 = \frac{7m^2}{32\pi f_\pi^2} = 0.156$. At present a_0 has been worked out to two loops in the chiral expansion (J.Gasser, H.Leutwyler;1982, J.Gasser et al.1996) and its value is predicted with unusual for strong interaction precision:
$a_0 = 0.22 \pm 0.005$
The values of S-wave scattering lengths a_0, a_2 are of particular interest since they vanish in the chiral limit of zero quark masses. Moreover their values are connected with the size of quark-antiquark vacuum condensate.

Determination of pionium ($\pi^+\pi^-$-atom) lifetime. Pionium is unstable through charge exchange reaction $\pi^+\pi^- \rightarrow \pi^0\pi^0$.

$\Gamma_{atom} = \frac{1}{\tau} \approx \alpha^3 \mid a_0 - a_2 \mid^2$

The primary goal: Determination of pionium lifetime with $\approx 10\%$ accuracy gives for $a_0 - a_2 \approx 5\%$

At present B.Adeva et al., Phys. Lett., 2005:

$\tau = [2.91 \begin{smallmatrix} +0.49 \\ -0.62 \end{smallmatrix}] \times 10^{-15} s; \quad \mid a_0 - a_2 \mid = 0.264 \begin{smallmatrix} +0.033 \\ -0.020 \end{smallmatrix}$

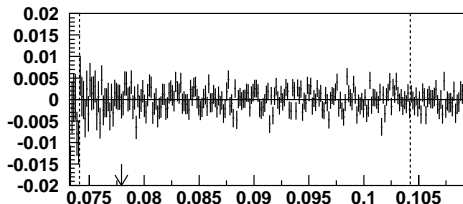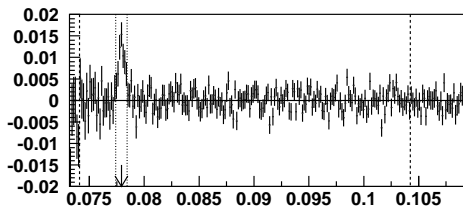Scattering lengths from NA48/2(J. Batley et al. EPJ, 2009):

$a_0 - a_2 = 0.2571 \pm 0.0087$

(N.Cabbibo, G.Isidori ,JHEP, 2005, J.Gasser et al. Phys.Lett., 2006)

$\Delta = \frac{(data - fit)}{data}$ vs M^2 in Cabibbo -Isidori approach.
a)Without pionium; b) with pionium

Higher order terms and Electromagnetic effects in $K^+ \longrightarrow \pi^0\pi^0\pi^+$

1.Under threshold $M_{\pi\pi} < 2m_c$: Pionium atom just under threshold.

2.Above threshold $M_{\pi\pi} > 2m_c$: Gamov-Sommerfeld factor.

Two unsolved challenges (2006) :

1.Higher order terms.

2.Electromagnetic effects.

To account for pionium effect NA48/2 multiplied the content of the bin centered at $M^2 = 4m_c^2$ by factor $(1 + f_{atom})$. The best fit value for $f_{a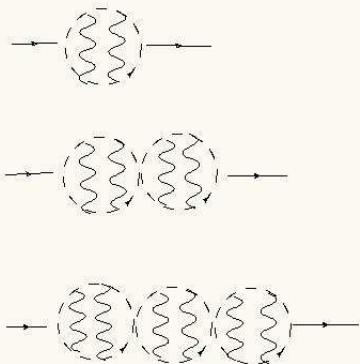tom}$ corresponds to probability of pionium atom production $P = \frac{K^+ \to \pi^+ atom}{K^+ \to \pi^+\pi^-\pi^+} \approx (1.69 \pm 0.29) \times 10^{-5}$, whereas the theory predict (Silagadze, JETP Letters, 1994) $P = 0.8 \times 10^{-5}$

Unstable atoms and electromagnetic effects

Double sum of strong loop with infinite photon exchanges

Strong interactions to all orders in $K \to 2\pi$

$$M_c(K \to \pi^+\pi^-) = \int \Psi_c^+(r)M_0(r)d^3r; \ k_c = \frac{\sqrt{M^2 - 4m_c^2}}{2}$$

$$M_n(K \to \pi^0\pi^0) = \int \Psi_n^+(r)M_0(r)d^3r; \ k_n = \frac{\sqrt{M^2 - 4m_n^2}}{2}$$

$$-\Delta\Psi_c(r) + U_{cc}\Psi_c(r) + U_{cn}\Psi_n(r) = k_c^2\Psi_c(r);$$
$$-\Delta\Psi_n(r) + U_{nn}\Psi_n(r) + U_{nc}\Psi_c(r) = k_n^2\Psi_n(r)$$

S-wave scattering, strong potential with sharp boundary $U_{ik} \gg k_{c(n)}$, asymptotic behavior of wave functions, unitarity constrains.

Final result. Strong $\pi\pi$ interaction to all orders.

The decay amplitudes $K \to 2\pi$ are expressed through the "unperturbed" decay amplitudes M_{0c}, M_{0n} and $\pi\pi$ scattering amplitudes

$$f_x(\pi^+\pi^- \to \pi^0\pi^0), \quad f_c(\pi^+\pi^- \to \pi^+\pi^-), \quad f_n(\pi^0\pi^0 \to \pi^0\pi^0)$$

$$M_c = M_{0c}(1 + ik_c f_{cc}) + ik_n M_{0n} f_x; \, f_{cc} = \frac{a_{cc}(1 - ik_n a_{nn}) + ik_n a_x^2}{D}$$

$$M_n = M_{0n}(1 + ik_n f_{nn}) + ik_c M_{0c} f_x; \, f_{nn} = \frac{a_{nn}(1 - ik_c a_{cc}) + ik_c a_x^2}{D}$$

$$f_x = \frac{a_x}{D}; D = (1 - ik_c a_{cc})(1 - ik_n a_{nn}) + k_n k_c a_x^2$$

In the case of exact isospin symmetry in the vicinity of threshold:

$$a_x = \frac{a_0 - a_2}{3}; \quad a_{nn} = \frac{a_0 + 2a_2}{3}; \quad a_{cc} = \frac{2a_0 + a_2}{6}$$

Electromagnetic interaction in $\pi^+\pi^-$ system

S.Gevorkyan,A.Tarasov,O.Voskresenskaya, Phys. Lett. 2007
Receipt is known for many years (Wigner, 1948, Breit, 1957,
Baz , 1957) $ik_c \rightarrow \tau = \frac{d\log[G_0(kr)+iF_0(kr)]}{dr}\bigg|_{r=r_0}$ F_0, G_0 are the
regular and irregular solutions of the Coulomb problem
(Hypergeometric functions). In the region $kr_0 \ll 1$:

$$\tau = ik - \alpha m\left[\log(-2ikr_0) + 2\gamma + \psi(1 - i\xi)\right]$$

$$Re\,\tau = -\alpha m\left[\log(2kr_0) + 2\gamma + Re\,\psi(1 - i\xi)\right], \xi = \frac{\alpha m}{2k}$$

$$Im\,\tau = kA^2, \quad A = \exp\left(\frac{\pi\xi}{2}\right)|\Gamma(1 + i\xi)|$$

$\gamma = 0.5772$; digamma function $\psi(\xi) = \frac{d\log\Gamma(\xi)}{d\xi}$

Final results. Electromagnetic interaction between pions to all orders.

1.Under threshold $M_{\pi\pi} < 2m_c$: Pionium atom just under charged pions threshold.

S.Gevorkyan,D.Madigozhin,A.Tarasov,O.Voskresenskaya, Phys.Part.Nucl. Lett.,2008

The contribution of pionium belong to the central bin and coincides with Silagadze prediction 0.8×10^{-5}. The excess in NA48/2 is connected with $\pi\pi$ proper electromagnetic interaction.

2.Above threshold $M_{\pi\pi} > 2m_c$: Gamov-Sommerfeld factor

$G = \frac{2\pi\omega}{1-e^{-2\pi\omega}}; \quad \omega = \frac{\alpha}{v}$

Decay with two charged pions in final state: $K^{\pm} \rightarrow \pi^{+}\pi^{-}e^{\pm}\nu$ (K_{e4} decay)

(Rosselet et al.,CERN ,1977 ($a_0 = 0.26 \pm 0.05$); Pislak et al. BNL ,2003 ($a_0 = 0.216 \pm 0.017$))

NA48/2,CERN Bloch-Devaux, Kaon workshops: December 2006; May,September 2007 $\sim 10^6$ events

$a_0^0 = 0.256 \pm 0.006 \pm 0.005$

ChPT $a_0^0 = 0.22 \pm 0.005$

1.The isospin symmetry breaking effects in K_{e4} decays.

2.The electromagnetic effects in K_{e4} decay.

Short introduction to $K^{\pm} \rightarrow \pi^+\pi^- e^{\pm} \nu$ decay

$$M = \frac{G_F \sin\theta_c}{\sqrt{2}} \langle \pi^+\pi^- | J_{had}^{\mu} | K^+ \rangle \langle e^+ \nu_e | J_{\mu}^{lep} | 0 \rangle.$$

$$F = f_s e^{i\delta_0^0(s)} + f_p e^{i\delta_1^1(s)} \cos\theta_{\pi}; \quad G = g_p e^{i\delta_1^1(s)}; \quad H = h_p e^{i\delta_1^1(s)}$$

$\delta_0^0(s), \delta_1^1(s)$-phases in $\pi\pi$ scattering (I=l=0 and I=l=1)

Fermi-Watson theorem: $\delta_0^0(s); \delta_1^1(s) \rightarrow \delta(\pi^+\pi^- \rightarrow \pi^+\pi^-)$

$\tan\delta_l^I = \sqrt{1 - \frac{4m_c^2}{s}} q^{2l} [A_l^I + B_l^I q^2 + C_l^I q^4 + D_l^I q^6]$

$q^2 = \frac{s - 4m^2}{4}$

Roy equations connect these coefficients with scattering lengths:

$A_l^I = A_l^I(a_0^0, a_0^2)....$

From experiment the difference $\delta = \delta_0^0 - \delta_1^1$ is extracted.

Charge exchange in K_{e4}

S.Gevorkyan, A.Sissakian, A.Tarasov, H.Torosyan, O.Voskresenskaya Phys.Atom.Nucl.,2010; hep-ph/0704.2675
We take into account the charge exchange (a la N.Cabibbo)
$K \to \pi^0\pi^0 e\nu \to \pi^+\pi^- e\nu$ For dipion state I=l=0

$$
\begin{aligned}
M &= M_c(1 + ik_c a_c) + ik_n a_x M_n; \; a_c = \frac{2a_0 + a_2}{3}(1 + \epsilon); \\
a_x &= \frac{1}{3}(a_0 - a_2)(1 + \epsilon/3); \; \epsilon = \frac{m_c^2 - m_n^2}{m_c^2}
\end{aligned}
$$

$$
\delta_0^0 = \arctan \frac{k_n a_n + k_c a_c}{1 + k_n k_c (a_x^2 - a_n a_c)} - \arctan k_n\left(a_n - \frac{1}{\sqrt{2}} a_x\right)
$$

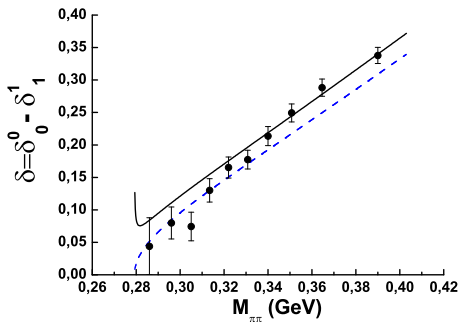# Modified phases by electromagnetic effects

Strong δ_{str} and electromagnetic δ_{em} :

$$\tilde{\delta}_0^0 = \delta_{str} + \delta_{em} \quad \delta_{str} = \arctan(A\tan\delta_0^0 + B\tan\delta_0^2);$$

$$\delta_{em} = \arctan(\frac{\alpha}{\beta}); \quad \beta = \frac{\sqrt{1-4\upsilon}}{1-2\upsilon}$$

$$A = \frac{2G(1+\epsilon) + \lambda(1+\frac{\epsilon}{3})}{3}; \quad B = \frac{G(1+\epsilon) - \lambda(1+\frac{\epsilon}{3})}{3}$$

$$G = \frac{2\pi\omega}{1-e^{-2\pi\omega}}; \quad \omega = \frac{\alpha}{\upsilon}$$

$$\lambda = \sqrt{\frac{1-4u_0}{1-4u_c}} \quad u_c = \frac{m_c^2}{s}; \quad u_0 = \frac{m_n^2}{s}$$

$$\tilde{\delta}_1^1 = \arctan\left(G(1+\frac{\alpha^2}{\beta^2})\tan\delta_1^1\right). \tag{1}$$

Modified and usual phases dependence on invariant mass

Conclusions

1. In the framework of non-relativistic quantum mechanics the relations between the kaons decay amplitudes to two pions with $\pi\pi$ final states interaction and without it (unperturbed) are obtained to all order in strong interaction.

2. The proposed approach allows one to correctly account for electromagnetic interaction between pions obtaining the bound states under the charged pions threshold (pionium atoms) and Gamov- Sommerfeld factor above it.

3. Accounting for isospin breaking effects in $K^{\pm} \rightarrow \pi^{+}\pi^{-}e^{\pm}\nu$ decays allowed to make agreement between scattering length value extracted from experimental data and chiral theory prediction.