THE EVOLUTION OF THE UNIVERSE

HOW DO WE KNOW TEMPERATURE AT TIME =t ?
ONE FINDS T(t) USING EINSTEIN'S THEORY−UGH!

	t = Time	T = Temperature	Events
Elementary particles	10^{-35} s	10^{14} GeV	Big Bang, Strings, Inflation Very early. Current particle theory no good
	10^{-11} s	100 GeV	Electroweak Phase Transition Particles (Higgs) get masses. Particly theory ok. Baryogenesis? (more particles than antiparticles)
	10^{-6} s		Start of QCD phase transition
Hadronic particles	10^{-5} s	100 MeV	QCD (quark−hadron) phase transition Quarks(elementary) condense to Protons
	1−100 s	1.0X 10^{9} ^{o}K	Nucleosynthesis: Helium, light nuclei formed Superconducting Universe
	380,000 years	0.25 ev, 3,000 ^{o}K	Atoms (electrically) neutral Last scattering of light (electromagnetic radiation) from big bang: Cosmic Microwave Background
	1 billion years		early galaxies form
	14 billion years	2.7 ^{o}K	Now

FROM EINSTEIN'S GENERAL THEORY OF OF RELATIVITY WE ALSO FIND R(t), THE RADIUS OF THE UNIVERSE AT TIME = t

3. TODAY WE STUDY BASIC ASTROPHYSICS CONCEPTS:

WHAT DOES ONE MEAN BY THE RADIUS OF THE UNIVERSE?

WHAT DOES ONE MEAN BY THE TEMPERATURE OF THE UNIVERSE

HOW DO THE RADIUS AND TEMPERATURE OF OUR UNIVERSE DEPEND ON TIME

WHAT CAN WE LEARN ABOUT THE UNIVERSE BY KNOWING THE TEMPERATURE AT DIFFERENT TIMES.

WHY IS THERE A UNIFORM TEMPERATURE (ALMOST) IN ALL PARTS OF OUR UNIVERSE

WHAT NEW CONCEPTS DOES RELATIVITY (SPECIAL AND GENERAL) INTROUDCE.

IS RELATIVITY CONSISTENT WITH QUANTUM MECHANICS. BOHR VS EINSTEIN

TIME, TEMPERATURE, RADIUS OF UNIVERSE USING FRIEDMANN'S EQUATIONS: Our approach:

1. Review NEWTON'S LAW OF MOTION

2. Discuss EINSTEIN'S SPECIAL AND GENERAL THEORIES OF RELATIVITY

3. Give FRIEDMAN'S EQUATION, which we need to find R(t) and T(t), the radius and temperature of the universe as functions of time (t), and to understand the concept of the RADIUS OF THE UNIVERSE

4. There is a problem with the concept of the temperature of the universe: Why is there a single temperature, and not different temperatures in different parts of the universe. Solution: INFLATION

NEWTON'S LAW OF MOTION, FORCE OF GRAVITY, AND ACCELERATION DUE TO THE FORCE OF GRAVITY: NONRELATIVISTIC

VELOCITY (SPEED) IS THE TIME RATE OF CHANGE OF POSITION ($r(t) \equiv \vec{r}(t)$ is a vector):

v(t)= distance/time =$\dot{r}$(t)

ACCELERATION IS THE TIME RATE OF CHANGE OF VELOCITY ($v(t) \equiv \vec{v}(t)$, $a(t) \equiv \vec{a}(t)$ are vector functions of t) :

a(t) = $\dot{v}$(t) =$\ddot{r}$(t)

IF m IS THE MASS OF A PARTICLE BEING CONSIDERED, AND F IS THE FORCE BEING EXERTED ON THE PARTICLE

NEWTONS LAW OF MOTION:

F = MASS $\times$ ACCELERATION, or F = m $\times$ a.

Therefore, $\ddot{r}$(t) = F/m .

MOMENTUM, ANOTHER IMPORTANT CONCEPT

NEWTONIAN MOMENTUM (p):

p=mv, nonrelativistic

ANOTHER FORM OF NEWTONS LAW:

F=$\dot{p}$= TIME RATE OF CHANGE OF p.

NEWTON'S LAW OF MOTION FOR THE FORCE OF GRAVITY-A REVIEW

F(GRAVITY) = FORCE OF GRAVITY. IF MASS M IS AT A DISTANCE R FROM OUR MASS m,

F(GRAVITY) = G $\frac{mM}{R^2}$ **, with**

G = Newton's gravitational constant, and m feels the force of gravity in the direction of M (attractive force)

THEREFORE NEWTON'S LAW OF MOTION FOR GRAVITY IS:

m a = G $\frac{mM}{R^2}$ **But a=** $\ddot{r}$

Therefore $\ddot{r} = G\frac{M}{R^2}$

is the acceleration of gravity at a distance R from a mass M in the direction toward M.

Example: M=M_e **= mass of earth, m is near the surface of the earth so R=**R_e **= radius of earth. Then a=g=acceleration of gravity**

g= G $\frac{M_e}{R_e^2} \simeq 9.8m/s^2 = 980cm/s^2$**, at the surface of the earth, with small variation depending whether you are in Pittsburgh or Greenland or ...**

EINSTEIN: SPECIAL THEORY OF RELATIVITY

EINSTEIN'S POSTULATES:

1) THE SPEED OF LIGHT=c IS THE SAME IN ANY INERTIAL (NOT ACCELERATED) VACUUM FRAME

2) THE LAWS OF PHYSICS ARE THE SAME IN ALL INERTIAL FRAMES

THESE POSTULATES RESULT IN IMPORTANT DIFFERENCES FROM NEWTONIAN THEORY:

Time and Distance in a moving system:

Let L be a length (say of a rod) and t a time interval (measured by a clock) in a system at rest, e.g., a train with no velocity.
If the train is passing you with speed u, you are at rest, and you measure the length of the rod (L') and the time interval of the moving clock (t') on the trian, Einstein's Special Theory of Relativity gives

$$L' = L\sqrt{1 - u^2/c^2} \quad \text{(length contraction)}$$

$$t' = t/\sqrt{1 - u^2/c^2} \quad \text{(time dilation)}$$

ADDITION OF VELOCITIES. If you are moving with velocity v_1 on something (say you are on a train) moving with velocity v_2 with respect to the stationary ground, with v_1, v_2 in the same direction, your velocity w.r.t. the ground is:

$$V = \frac{v_1 + v_2}{1 + v_1 v_2 / c^2}$$

For example, if both v_1 and v_2 are much smaller than c, $V = v_1 + v_2$, the same as in Newtonian theory.

Another example, if $v_1 = v_2 =$c (both you and the train are moving with the speed of light), then V=2c/2=c, which is consistent with Einstein's first postulate.

ALSO, FROM THIS WE FIND ANOTHER IMPORTANT ASPECT OF THE SPECIAL THEORY OF RELATIVITY: INFORMATION CANNOT BE SENT FASTER THAN THE SPEED OF LIGHT.

LORENTZ TRANSFORMATION: SPACE AND TIME IN A MOVING FRAME. If position, time are (x,y,z,t) in a rest frame, (x',y',z',t') in a system moving with velocity u in the x direction are:

x'=(x-ut)/$\sqrt{1 - u^2/c^2}$, y'=y, z'=z,

t'=(t-ux/c^2)/$\sqrt{1 - u^2/c^2}$.

Definition of Momentum in Special Relativity:

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}}$$

Newton's Law of Motion becomes, with Einstein's momentum

$$\vec{F} = \frac{d\vec{p}}{dt}$$

Energy = kinetic + potential + mass energy

mass energy = m c^2

Energy of a free particle = $\sqrt{p^2c^2 + m^2c^4}$ **with p = momentum of the particle (even if it has no mass (a photon)). If m=0 (like a photon), E=pc.**

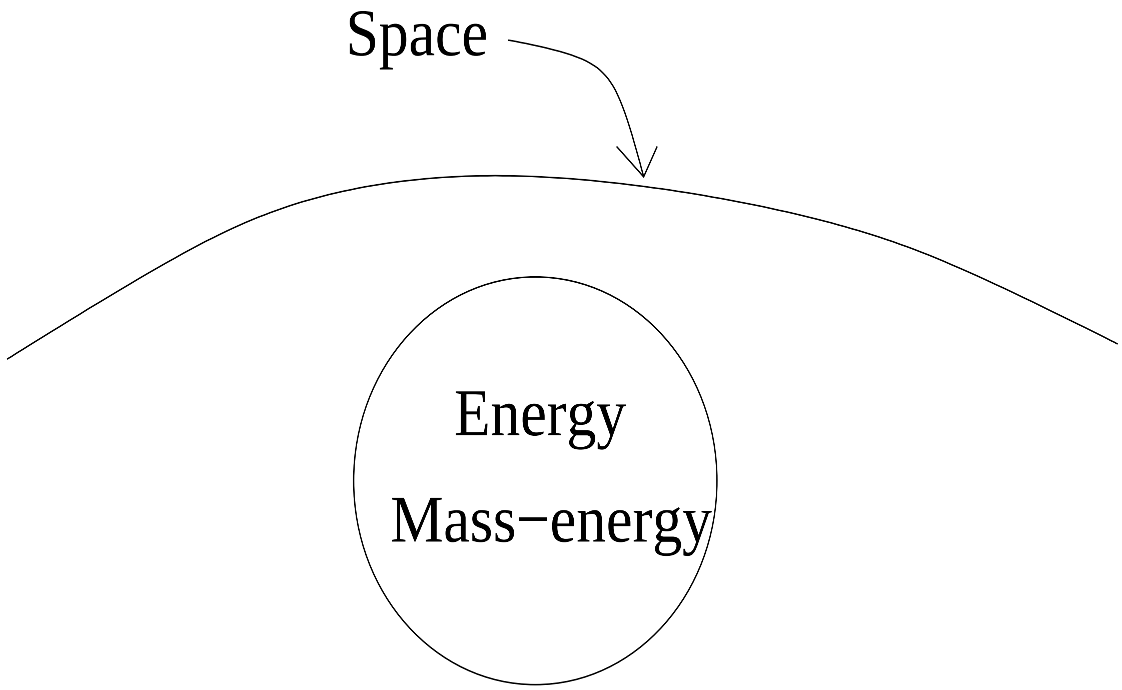
Einstein's laws of motion are the same as Newton's except for the axioms about the speed of light and the definition of momentum. If objects have mass and move slowly compared to c Einstein's and Newton's laws of motion are almost the same

HOWEVER, THE IMPORTANT RESULT FROM EINSTEINS'S SPECIAL THEORY OF RELATIVITY, THAT INFORMATION CANNOT BE TRANSFERRED FASTER THAN THE SPEED OF LIGHT DEFINES THE RADIUS OF THE UNIVERSE (DISTANCE FROM US TO THE EDGE OF OUR UNIVERSE):

$\dot{R}(t)$**=SPEED OF RADIUS OF UNIVERSE = c DEFINES R(t) FOR AN EXPANDING UNIVERSE (LIKE OURS). An object at distance r>R(t) is out of causal contact (ala Einstein)**

EINSTEIN: GENERAL THEORY OF RELATIVITY: Energy (mass), space, and time related by the equations of general relativity. Essential for understanding the very early universe, black holes, ... Quite complicated!

ONE ESSENTIAL NEW CONCEPT IS THAT SPACE CAN BE CURVED



k = curvature

[hot news: k=0 = flat space]

[space "almost flat"−to be discussed]

FRIEDMANN'S EQUATION FOR R(t)=RADIUS OF SURFACE OF UNIVERSE

See Kolb/Turner, The Early Universe

Einstein's equations of general relativity predicted that the universe either expanded or contracted. Before Hubble showed that it expanded, Einstein added a COSMOLOGICAL CONSTANT, Λ. After Hubbles observations, Einstein said that "IT WAS MY GREATEST BLUNDER". After dark energy was discovered, Λ has returned –we'll study dark energy when we do early universe phase transitions. Here we assume that Λ=0.

Friedman's equations for R(t)=radius of the universe, $\ddot{R}$ = acceleration of R(t), are:

Friedmann's Equation:

$$\frac{\ddot{R}(t)}{R(t)} = -\frac{4\pi G}{3}(\rho + 3p)$$

$$\rho = \text{energy density} = \text{energy/volume}$$

$$p = \text{pressure} = \text{force/area}$$

Friedman also has an equation involving the constant k=curvature of space, which we do not need

We shall now see that Einstein's equation of general relativity in Friedman's simple form is similar to that of Newton centuries earlier.

COMPARISON BETWEEN FRIEDMAN'S FIRST EQUATION AND NEWTONS LAW OF MOTION

Newton's Law of Motion for acceleration due to gravity force, using F(gravity) on mass m a distance R from a mass M=mMG/R^2=m x acceleraton:

$$\ddot{R}(t) \;=\; \text{acceleration of force of gravity towards M} = \frac{MG}{R^2}\;,$$

with M=mass = density x volume = $\rho \times \frac{4\pi}{3}R^3$.

Therefore, with the direction towards M being inward (negative)

$$\begin{aligned}\ddot{R}(t) &= -\frac{4\pi}{3}\frac{R^3}{R^2}G\rho \\ &= -\frac{4\pi}{3}G\rho R \\ \frac{\ddot{R}(t)}{R(t)} &= -\frac{4\pi}{3}G\rho\;,\end{aligned}$$

which is Friedman's equation without pressure.

Recall definition, p=pressure =force/area

Note, force x distance = work = energy.

If the force is exerted through a distance = d,

p=force/area =force x d/(area x d)=energy/volume, as area x distance = volume

Therefore ρ+3p in Friedman's equation includes mass energy + energy due to force.

CRITICAL DENSITY AND CURVATURE OF SPACE

Critical Density: $\rho = \rho_c$ **for the universe to neither collapse nor continue expansion forever**

Closure Parameter:

$$\Omega \;=\; \frac{\rho}{\rho_c} = 1 + k\frac{c^2}{H^2 R^2}\;,$$

with H = Hubbles constant and R = radius of the universe.

Therefore $\Omega = 1.0$ **for k=0, flat universe**

The measurement of Ω **is one of the most important measurements for Cosmology.** Ω **is a measure of the total mass-energy in the universe.**

Ω **IS DETERMINED BY COSMIC MICROWAVE BACKGROUND RADIATION (CMBR)- TO BE DISCUSSED**

SOLUTIONS TO FRIEDMANN'S EQUATION FOR R(t) STARTING WITH THERMODYNAMICS

We use the first law of thermodynamics, with p=pressure, and the equation of state relating pressure to density

The first law of thermodynamics is:

$$\text{change in internal energy} \;=\; -p \times \;\text{change in volume}\;.$$

The equation of state for the universe can be written

$$p \;=\; w\rho\;,$$

where w is a constant relating pressure to density

Using the equation of state and the first law of thermodynamics one can show (for those mathematically inclined see the following page):

$$\rho \;\propto\; R^{-3(1+w)} \;\text{or}$$
$$\rho \;=\; constant \times R^{-3(1+w)}$$

Skip this page unless you like mathematical proofs.

Using internal energy of our system (the universe) = density x volume, and the volume of the universe = $(4\pi/3)R^3$**, the first law of thermo can be written (d ≡ change):**

$$\begin{aligned} d[\rho R^3] &= -pd[R^3] \ \text{ or using [d(xy) = xdy + ydx]} \\ d[(\rho + p)R^3] &= R^3 dp \end{aligned} \tag{1}$$

The equation of state for the universe can be written

$$p = w\rho \ . \tag{2}$$

EXERCISE: Using Eq(2) show that the solution to Eq(1) is

$$\begin{aligned} \rho &\propto R^{-3(1+w)} \text{ or} \\ \rho &= constant \times R^{-3(1+w)} \end{aligned}$$

[For those interested, one can prove that this is a solution to the thermodynamic equation, Eq(1), by using the rule

$$d[x^A] = Ax^{A-1}dx \ , \tag{3}$$

where A is a constant and x is the variable (which is R^3 **in our case).]**

The very early universe is dominated by electromagnetic radiation, and in a E-dominated system the radiation pressure vs radiation density (equation of state) is

$$\begin{aligned} pressure &= density/3 \text{ or} \\ p &= \rho/3 \text{ or} \\ w &= \frac{1}{3}\,. \end{aligned}$$

Thus using $\rho = constant \times R^{-3(1+w)}$ **, for the E (radiation) dominated universe**

$$\begin{aligned} \rho &\propto \frac{1}{R^4} \text{ and} \\ R(t) &\propto t^{1/2} = \sqrt{t} \end{aligned}$$

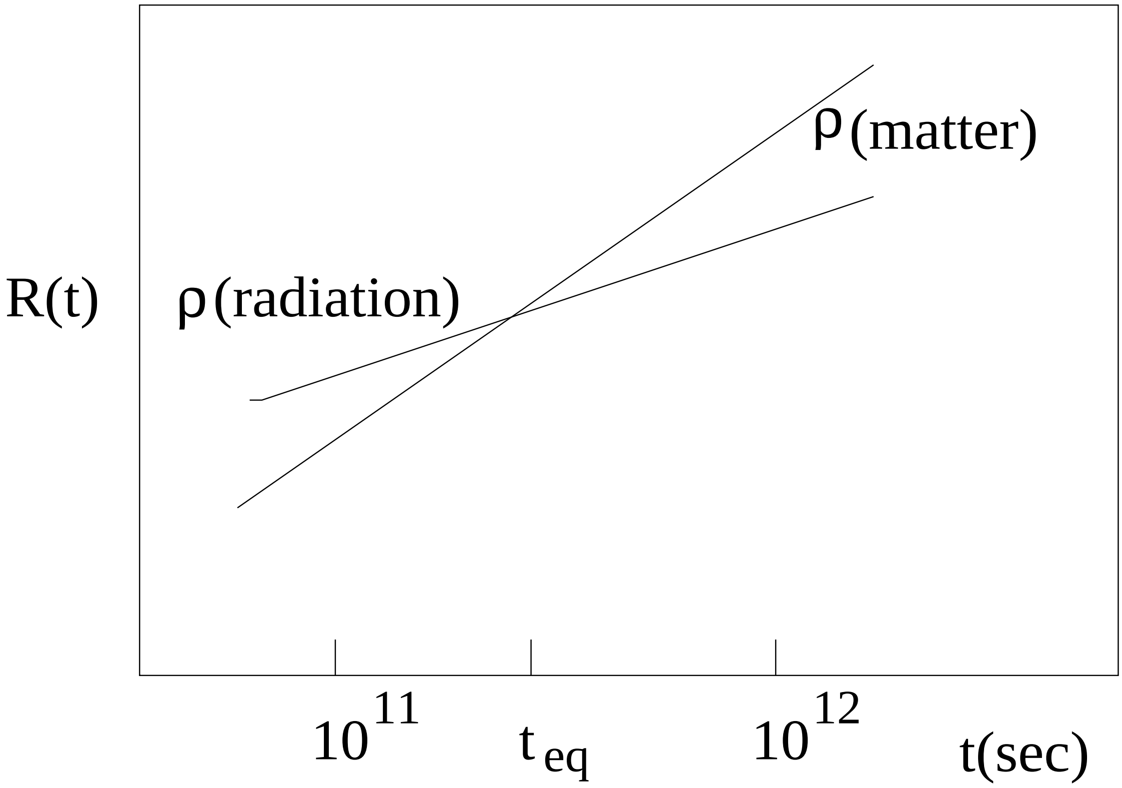
Matter dominated universe

$$\begin{aligned} pressure &= p = 0 \ \ \text{free matter, no pressure} \\ w &= 0 \end{aligned}$$

Solutions for density, radius, and time, matter dominated

$$\begin{aligned} \rho &\propto \frac{1}{R^3} \\ R(t) &\propto t^{2/3} \end{aligned}$$

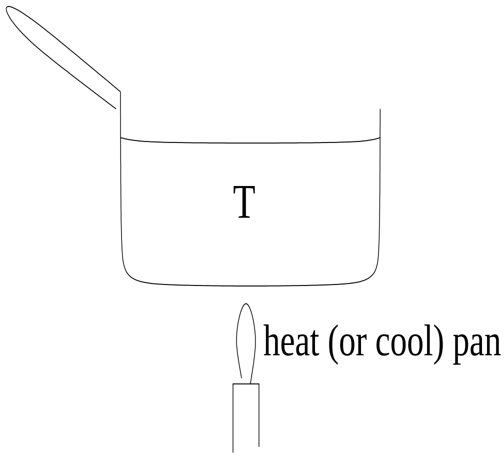
RADIATION−MATTER EQUILIBRIUM



$t_{eq} \simeq$ 1500 years

QUESTION: WHY DO WE WANT TO KNOW TEMPERATURE, T(t), AT DIFFERENT TIMES, t

ANSWER: WHAT IS MOST IMPORTANT FOR US IS THAT MATTER EXISTS IN DIFFERENT PHASES AT DIFFERENT TEMPERATURES. FOR EXAMPLE, ICE, WATER, STEAM



T < 0 C, ice

T=0 , T remains at 0 until all ice melts, becomes water
(Heat goes into latent heat, not increase in T)

T=100 , water starts turning into steam, T stays at 100
(More latent heat)

THE UNIVERSE ALSO HAS IMPORTANT DIFFERENT PHASES: A PHASE WITH NO PARTICLES HAVING MASS, A PHASE WITH ELEMENTARY PARTICLES WITH MASS, A PHASE WHERE THE QUARKS CONDENSE TO PROTONS, LIKE STEAM CONDENSING TO WATER DROPS. WE SHALL STUDY THESE PHASES IN SESSION 5

T(t) from Friedmann's Equation

Using the solutions to Einstein's/Friedmann's equations one can find the temperature, T (or energy E) at any time, t (See Kolb-Turner):

$$T(t) \simeq \frac{1\text{MeV}}{\sqrt{t(\text{in s})}} \, .$$

From this one finds:

t=10-100 trillionth s, T,E $\simeq$ 300-100 Gev $\simeq$ mass of Higgs. EWPT, particles get mass.

t=10-100 millionth s, T,E $\simeq$ 300-100 MeV $\simeq$ mass of pion $\simeq$ quark condensate, QCDPT, quarks condense to protons.

t=380,000 years, T,E $\simeq$ 0.25 ev. Atoms form, universe has electric charge $\simeq$ 0. Cosmic Microwave Background Radiation (CMBR) is released.

HORIZON: Since information cannot travel faster than the speed of light=c, the distance scale of our universe ("causally connected"), l_o is given at time t_o by

$$l_o \;=\; ct_o \;,$$

and at a very early initial time t_i the causally scale would have been

$$l_i \;=\; ct_i \;.$$

By using the presently known expansion rate of the universe we know the size of the homogeneous region, L_i from which our present universe originated at $t = t_i$, and

$$L_i \quad \text{is much greater than } \mathrm{l_i} \;.$$

This means that at early times the matter making up our present universe WAS NOT CAUSALLY CONNECTED. THIS IS THE HORIZON PROBLEM. HOW CAN WE HAVE A HOMOGENEOUS, ISOTROPIC UNIVERSE (as we'll see when we discuss the cosmis microwave background radiation-CMBR) IF IT IS MADE FROM A HUGE NUMBER OF DISCONNECTED REGIONS. SOLUTION: INFLATION

INFLATION:

Recall the Friedmann Eq. with no cosmological constant

$$\frac{\ddot{R}(t)}{R(t)} = -\frac{4\pi G}{3}(\rho + 3p)$$
$$\rho = \text{energy density} = \text{energy/volume}$$
$$p = \text{pressure} ,$$

and that $\ddot{R}(t)$ is the acceleration of the rate of expansion, given by Hubbles constant.

We expect that $\ddot{R}(t) < 0$. That is that the expansion is slowing down due to the pull of gravity. Since both ρ and p are expected to be positive, this is what Friedmann's equation says.

The current model of inflation says at a time about 10^{-34} sec the pressure was so negative that $\rho + p < 0$. By $t = 10^{-32}$ sec, still too early for us to use the standard model of elementary particles and fields, $\rho + p$ reversed sign, and inflation stopped. In this short time the expansion was so enormous that all of the regions that evolved for 14 billion years to form our universe were causally connected, and almost completely homogeneous and isotropic

INFLATION SOLVES THE HORIZON PROBLEM. ANY OBSERVATIONAL EVIDENCE FOR INFLATION. YES, AS WE SHALL SEE BY EXAMINING THE LATEST DATA ON THE CMBR

Example: Inflation due to cosmological constant, Λ

Friedmann's Eq with a comological constant is

$$\frac{\ddot{R}(t)}{R(t)} \ = \ -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \ . \tag{4}$$

In the model in which $p = -\rho = -\frac{\Lambda}{8\pi G}$ Eq(8) gives for the acceleration of the radius of the universe expansion rate of the universe is

$$\ddot{R}(t) \ = \ \frac{2\Lambda}{3} R(t) \ . \tag{5}$$

Eq(5) is that of a classical harmonic oscillator, but with the opposite sign of the expected $\ddot{r}(t) = -kr(t)$, which would describe a vibrating spring, with a spring constant k. taking two time derivatives on can prove that

$$R(t) = R(t = 0)e^{\sqrt{\frac{2\Lambda}{3}}t} \ . \tag{6}$$

I.e., with certain values of Λ there is an exponential expansion of more than 10^{26} in 10^{-34} sec in this model. A universe with a radius of 1 m would have a radius of 10^{26} m after inflation. Therefore our universe is homogenious. We shall learn that Λ, the Cosmological Constant, represents Dark Energy (sessions 4 and 5).

HOW BIG IS OUR UNIVERSE NOW?

What is the size of our universe now (at 14 billion years after the big bang). Let us use Hubbles law to estimate the radius of the universe now, R_0, by taking the velocity to be c (v=c at surface of universe):

$$\begin{aligned} c &= H_0 \times R_0 \\ R_0 &= \frac{c}{H_0} \end{aligned} \tag{7}$$

Using c= 300,000 km/s and H_0=71 km/(s-Mpc), with 1 pc= 3.25 ly $= 3 \times 10^{16} m$, **1 Mpc** $= 10^6 \times 3 \times 10^{16} m$ **we find**

$$R_0 \simeq 1.3 \times 10^{26} \text{ m} , \tag{8}$$

or $R_0 \simeq 1.4 \times 10^{10}$ **ly = 14 billion light years,**
as expected, since the universe is 14 billion years old

As we shall see next session, the universe is 14 billion years old. Since the radius was very tiny 14 billion years ago, it is reasonable that the causally connected universe now has a radius of 14 billion light years.

A.EINSTEIN vs N.BOHR, or

RELATIVITY vs QUANTUM MECHANICS

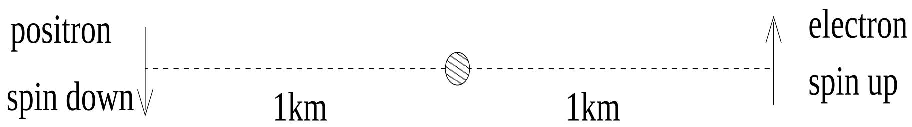
CONCEPTS:

ANGULAR MOMENTUM IS CONSERVED

ELECTRONS AND POSITRONS HAVE QUANTUM SPIN (ANGULAR MOMENTUM). THE SPIN CAN ONLY POINT UP OR DOWN

A SPIN ZERO PARTICLE DECAYS INTO AN ELECTRON AND A POSITRON. MOMENTUM CONSERVATION: THEY MOVE IN OPPOSITE DIRECTIONS

ANGULAR MOMENTUM =0: SPINS OPPOSITE



EINSTEIN: MEASURE ELECTRON, FIND SPIN UP. INSTANTANEOUSLY KNOW POSITRON SPIN DOWN

INFORMATION: 2 km IN ZERO TIME

RELATIVITY VIOLATED. QM WRONG!

BOHR: IT IS TRUE THAT THE POSITRON MUST HAVE SPIN DOWN, BUT THERE IS NO WAY TO DELIVER INFORMATION 2 km INSTANTANEOUSLY

BOHR: NO CONFLICT QM-RELATIVITY

VOTE BY PHYSICISTS: SORRY ALBERT, BUT NIELS BOHR IS CORRECT

NEXT SESSION: CMBR

Time = 380,000 years

Temperature $\simeq$ 0.25 eV

Electrons bind to atomic nuclei. Universe becomes electric charge neutral

Light from the early universe is released, the Cosmic Microwave Background Radiation (CMBR)

From CMBR (very complicated) studies we learn a lot about the universe